Statistical Instability of Barrier Micro-Discharges Operating in Townsend Regime

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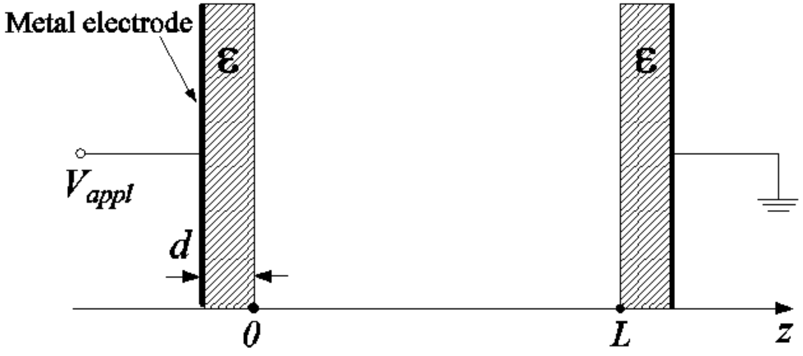
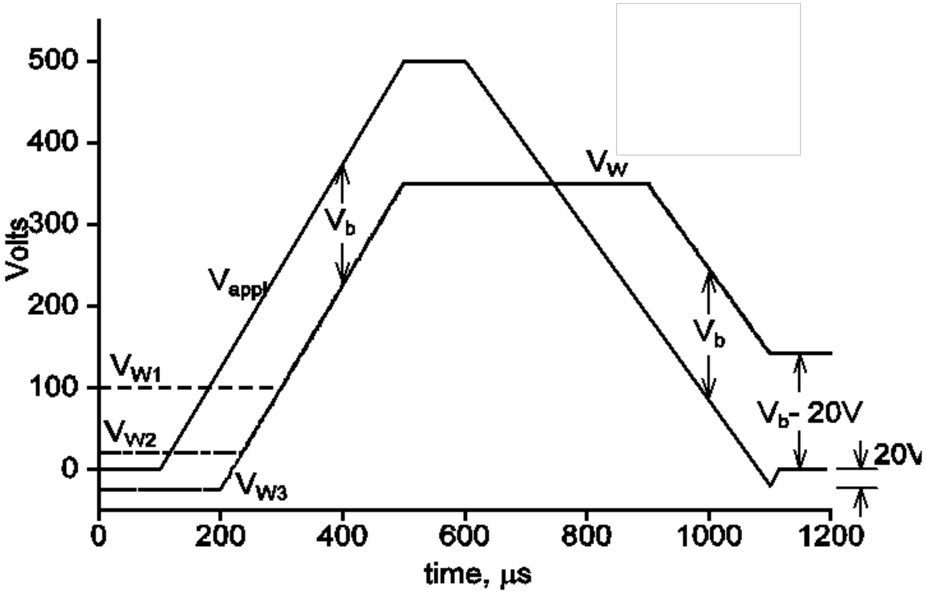
New kind of instability of a macroscopic physical system – statistical instability has been discovered when investigating a dielectric barrier discharge operating in a Townsend regime using 3D PIC/MC simulations. The dynamics of such discharge is studied analytically and via kinetic 3D PIC/MC simulations for the case of the ramp discharge in a plasma display panel (PDP) cell. It is shown that fluctuations of the number of charged particles in the discharge gap can be large; they strongly influence the dynamics of natural oscillations of the discharge current, and even lead to the disruption of the discharge. Unlike regular macroscopic instability, which grows exponentially with time, this instability works through random steps between natural oscillations, and discharge dies in one of the current minimums, where fluctuations are largest. Common view of ramp discharges based on multi-cell or time averaged measurements, and corresponding fluid or Boltzmann descriptions are inadequate. A simple model of the system is suggested to evaluate the level of fluctuations for different values of the discharge parameters (such as the current, secondary electron emission coefficient, dielectric capacitance, etc.). The role of external sources and particularly exoemission as a possible stabilizer of the ramp discharge in a PDP cell is clarified. Possible occurrence of such instability in a macro-system (plasma actuator) is presented.

DBD Townsend Discharge

- $I_0 = CdV/dt$

PDP Ramp (*L.F. Weber, 1998*)





$$\Delta_T \equiv \gamma(e^{\alpha L} - 1) - 1 = 0$$

α, γ - first and second Townsend coefficients

1D Fluid theory of the Ramp discharge (2000)

- ***Nagorny, Drallos, Weber (2000)***

$$|\Delta_T| << 1,$$

$$\tau_i \equiv L / v_i \;\; << \tau_V \sim CV_{br} / j \qquad (C \sim \varepsilon / (8\pi d))$$

$$\frac{\partial j}{\partial t} = \frac{\kappa}{L}(V - V_{br})\, j$$

$$\frac{\partial V}{\partial t} = -C^{-1}(j - j_{DC})$$

Stationary solution:

$$V = V_{br}\, ,$$

$$j(t) = j_{DC} \equiv C\, dV_{appl} / dt$$

1D Fluid theory of the Ramp discharge (2000)

- **Hamiltonian Formulation**

Variables: $p = V - V_{br}, \quad q = \ln(j / j_{DC})$

Hamiltonian equations:

$$\dot{q} = p/m = \frac{\kappa}{L} p = \frac{\partial H(p,q)}{\partial p}$$

$$\dot{p} = \lambda(1 - e^{q}) = -\frac{\partial U}{\partial q} = -\frac{\partial H(p,q)}{\partial q}$$

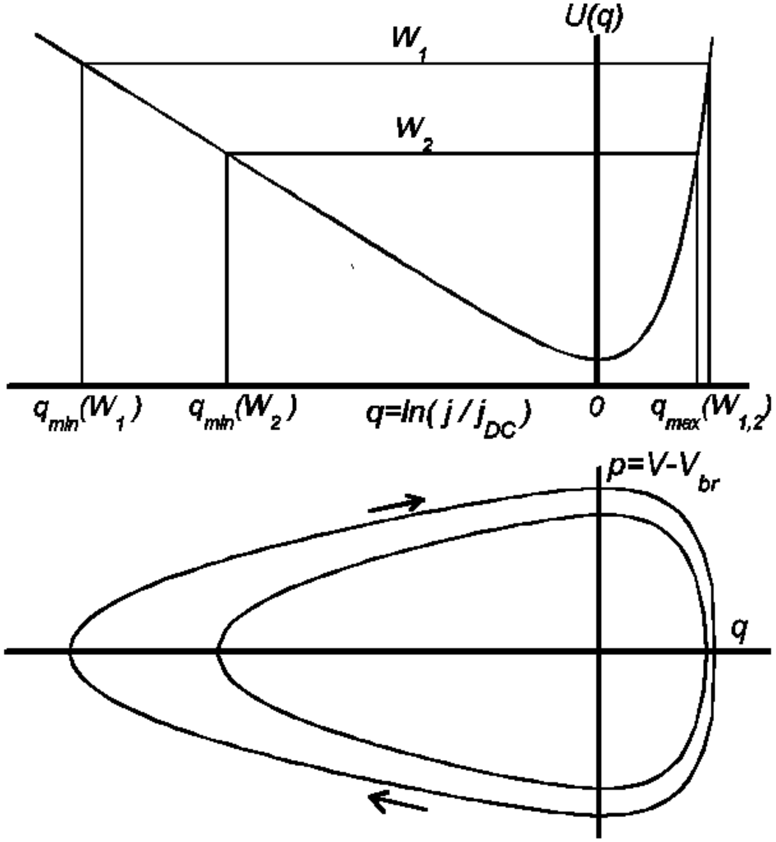
Hamiltonian:

$$H(p,q) = \frac{p^2}{2m} + U(q), \qquad U(q) = \lambda(e^{q} - q), \qquad \lambda \equiv dV_{appl}/dt$$

1D Fluid theory of the Ramp discharge (2000)

- **Integral of motion**

$$W = H(p,q) = const$$



- Periodic Oscillations

$$T = \oint \frac{dq}{\sqrt{(2\kappa / L)(W - U(q))}}$$

- Small amplitude, $j_{\max} \sim j_{DC}$

$$\omega_0 \sim \sqrt{(\partial\alpha / \partial V)\big|_{V_{br}} \lambda L / \tau_i}$$

- Large amplitude, $j_{\max} >> j_{DC}$

$$\omega_{large} \to \sim \sqrt{5 j_{DC} / j_{\max}}\ \omega_0$$

$$j_{\max} = j_{DC} \ln(j_{DC} / j_{\min})$$

1D Fluid theory of the Ramp discharge (2000)

- Additional sources of electrons/ions (metastables, exoemission,..) result in decay of oscillations

$$\dot{q} = \frac{\kappa}{L} p + \frac{e}{\tau_i j} \hat{S}_m$$

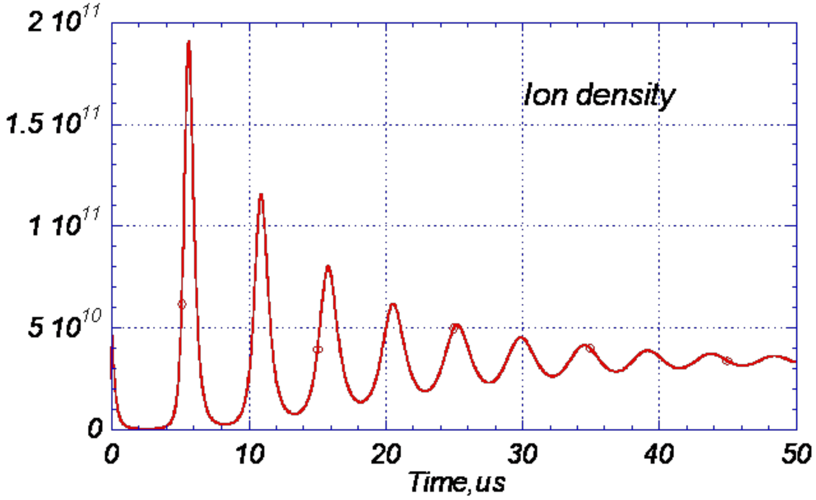
$$\frac{dW}{dt} = -\frac{e\hat{S}_m}{\tau_i} \frac{\dot{p}}{j}$$

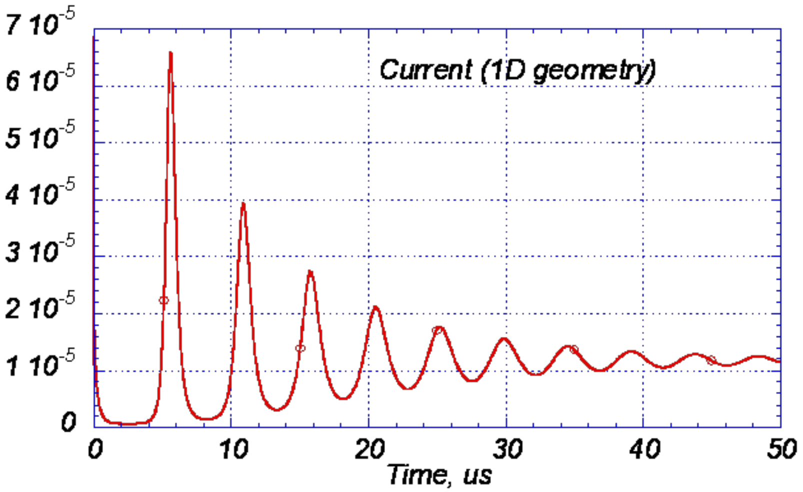
$$\delta W_t^{t+T} = -\frac{e\hat{S}_m}{\tau_i} \oint_{C(W)} \frac{dp}{j} < 0$$

- **Discharge stable if :**

  $dV/dt < \lambda_{max} (L, \gamma)$

  Good priming (W - small)

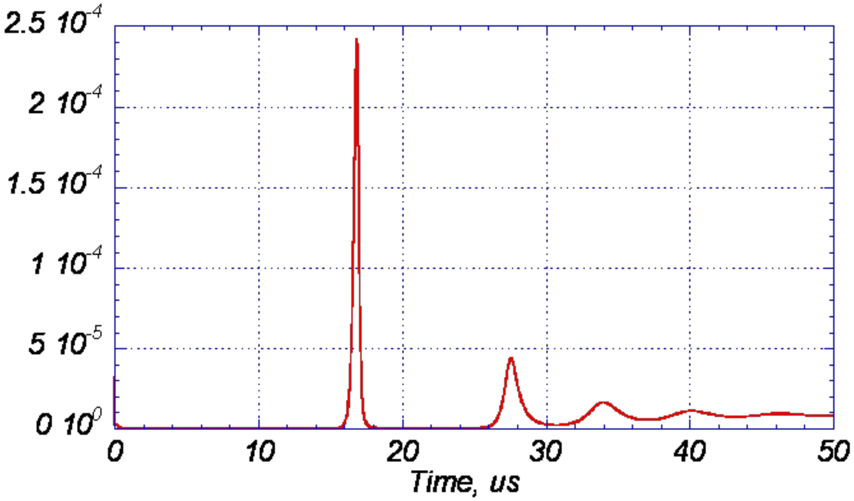


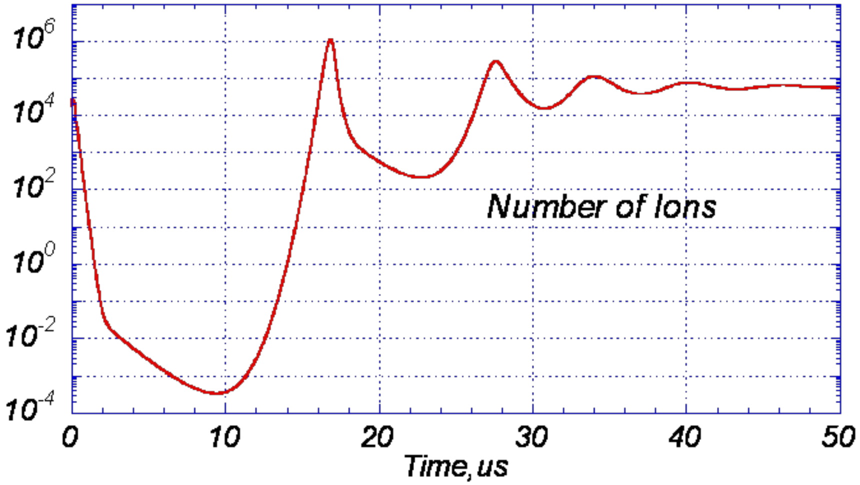


What’s the problem with fluid-like theory?

- PDP cell volume $\sim 10^{-5} cm^{-3}$ ➔
- Number of particles is not so large.

  In minimums it may become even less than 1



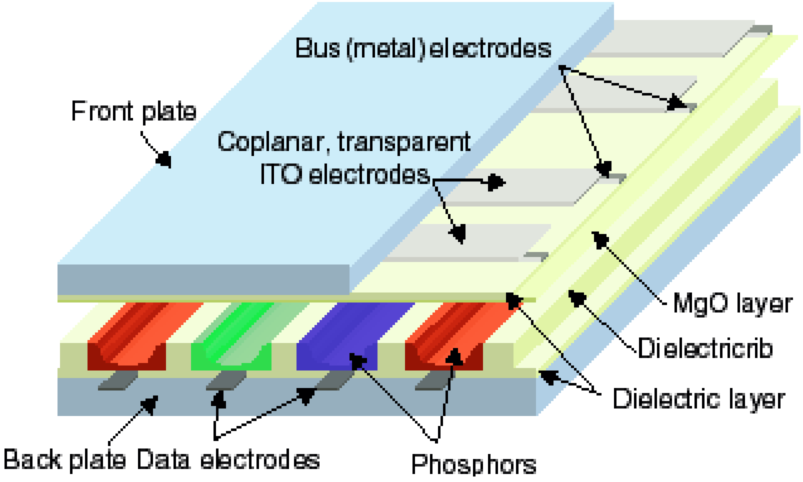


- Capacitance of the dielectric in a PDP cell ~ 0.02pF

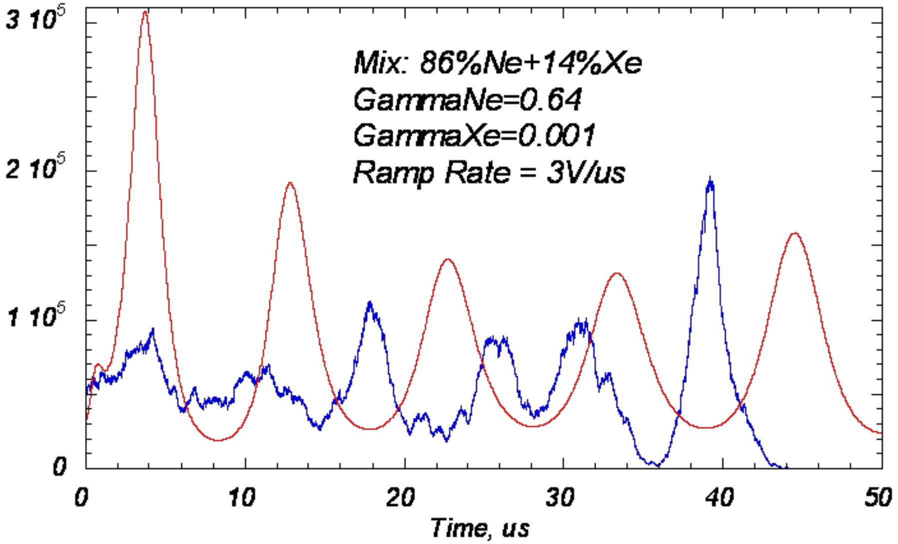
$\langle N_i \rangle \sim \tau_i (I_0 /2e)=(\tau_i /2e)CdV/dt \sim (1\text{-} 6)10^4$ - **Fluctuations may be important**

Ramp: 3D PIC/MC simulation vs. 3D Fluid

- *PDP cell*



Number of Ions in the cell

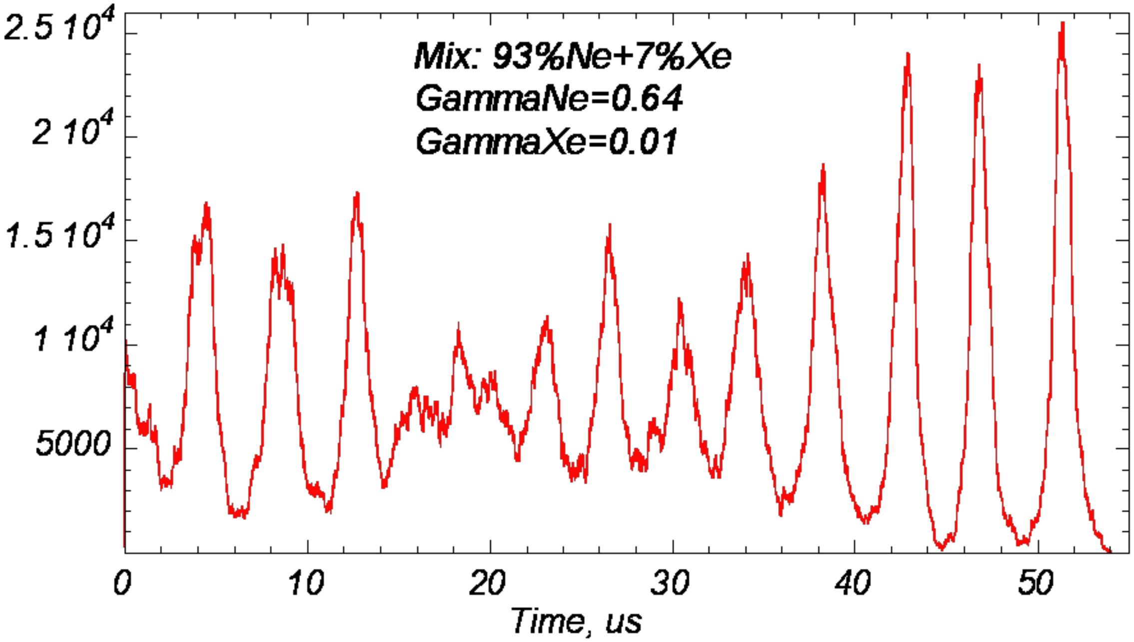


Red – 3D fluid
Blue – 3D PIC/MC

Ramp: 3D PIC/MC simulation of 1D cell

- *1D cell (reflective side walls), Ramp Rate = 3V/us,*
  $< N_i > \sim 10000$ (3D PIC/MC)

Number of Ions in the cell

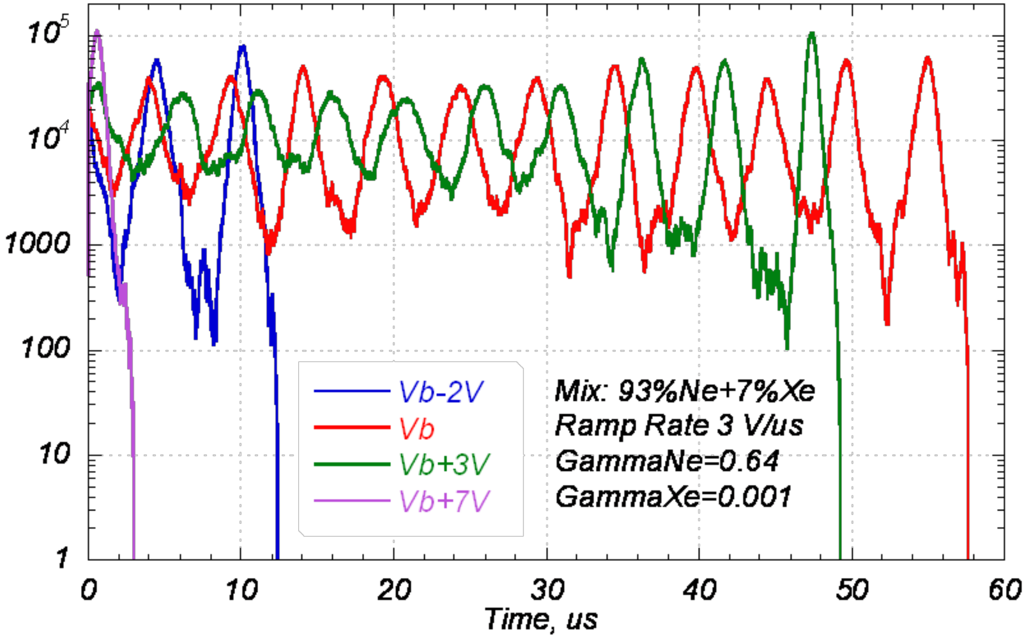


Instead of steady current (fluid) – large oscillations, and disruption.

Discharge "Lifetime" vs. initial conditions

- *1D cell, "Ideal" initial conditions,* $\langle N_i \rangle \sim 10^4$
- $V(t=0) \sim V_b$

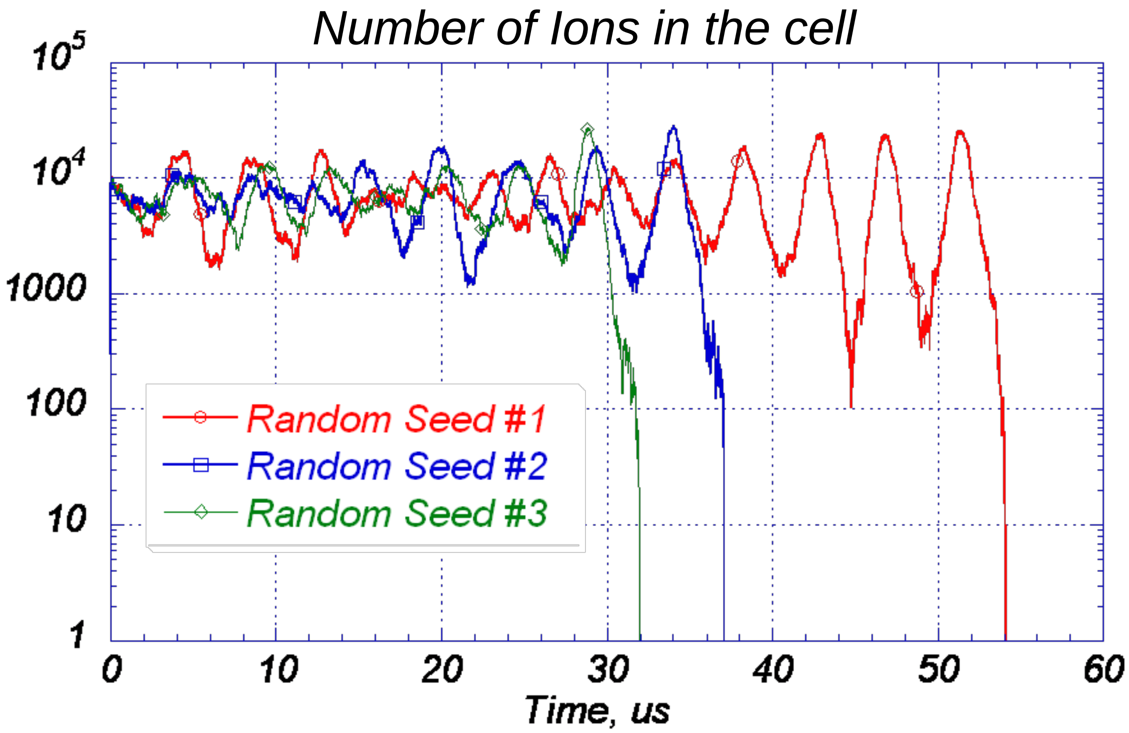
Number of Ions in the cell



Is this really a "lifetime"?

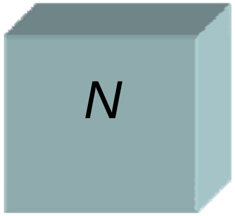
Ramp discharge “Lifetime” experiment

- *1D cell, < Ni > ~10000 (3D PIC/MC)*
- *C=0.016pF, Ramp rate = 4.2V/μs ;*
- *Everything identical except Random Seeds.*



Fluctuations

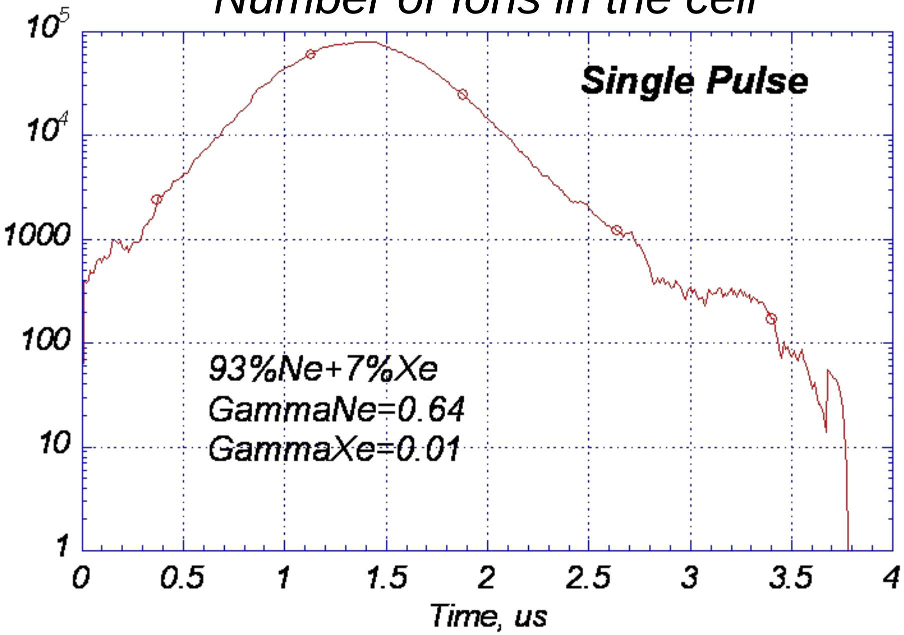
- Fluctuations in equilibrium
  $\delta N \sim N^{1/2}$



- For $< N_i > \sim 10000$,
  $\delta N_i \sim 100$ *(1%)* - fluid approximation seems good *(99.7% less than* 3σ*).*

What is wrong?

Number of Ions in the cell



Large deviations from fluid theory begin at N ~ 1000

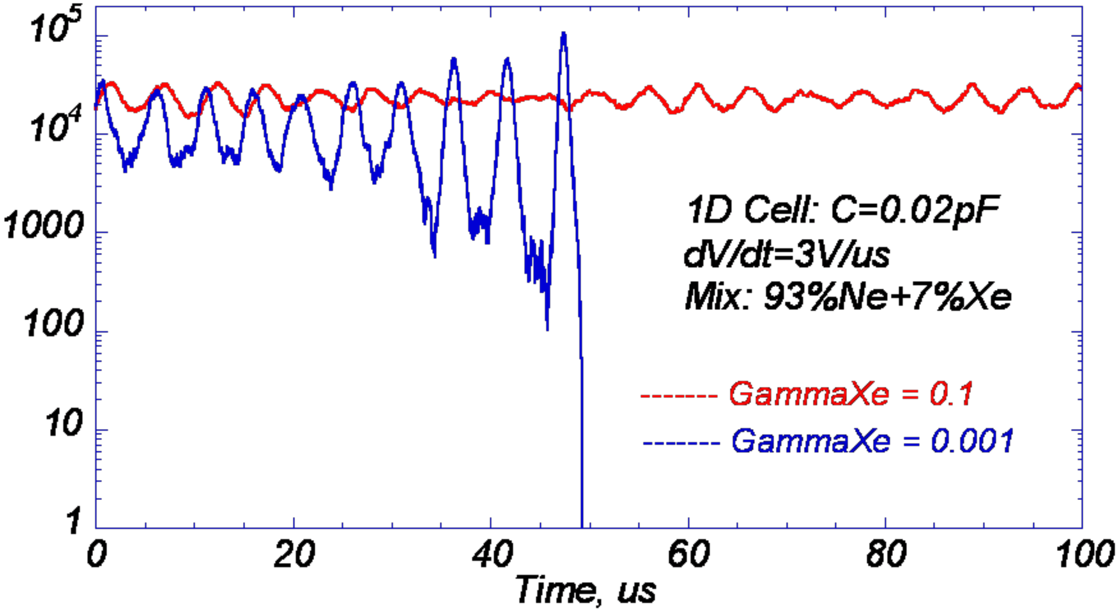
Fluctuations of the Ramp/Townsend discharge

- In the Townsend discharge N_i is the result of a balance, rather than equilibrium ($\delta N \sim N^{1/2}$):
  - ❑ $\boldsymbol{N_i \rightarrow N_e = \gamma N_i \rightarrow N_i = (\gamma N_i)\,\exp(\alpha L) = N_i}$
    - $N_e = \gamma N_i + (\gamma N_i)^{1/2} \rightarrow N_i + (N_i/\gamma)^{1/2}$ *(sec. emission)*
    - $N_e = \gamma N_i \rightarrow N_i + \delta N_i$ *(avalanche),* $\delta N_i \sim (N_i/\gamma)^{1/2}$
  - ❑ $\delta N_i \sim (N_i/\gamma)^{1/2} >> (N_i)^{1/2}$ ***in a single ion transit time***
- For the Ramp discharge PDP cell is statistically small: $<N_i> \sim 10^4\text{-}10^5$, $\gamma \sim 0.001\text{-}0.01$, $\delta N_i/N_i \sim 0.03\text{-}0.1$ *!!!*

  They are even larger in minimums!!!

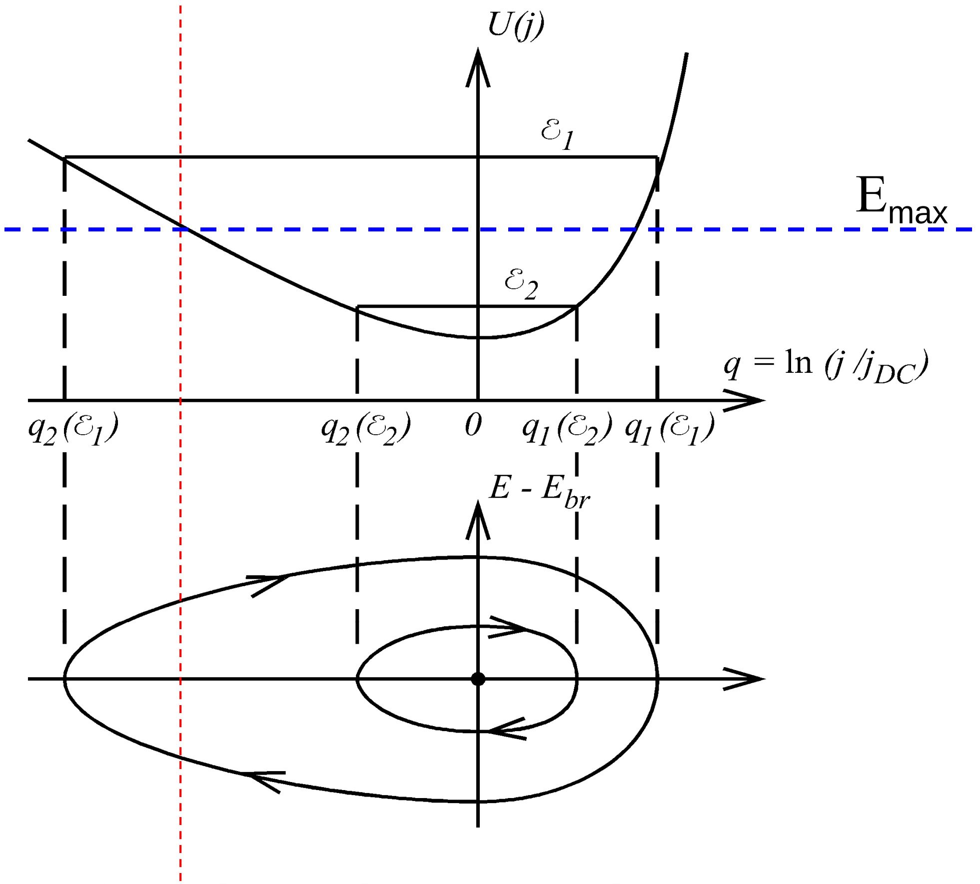
Dependence of fluctuations on γ (simulations)

- 1D cell, 3D PIC/MC : γ_{Xe}- dependence



Statistical Instability – how it works

Fluctuations lead to diffusion between “fluid phase trajectories”. Oscillations increase or decrease until large oscillation occurs. Fluctuations are very large (δN ~ N) in the minimums (N ~<1000). In one of minimums discharge dies.



- **Statistical Instability is powerful – it works even when** $<N_i>$ is large, or γ is not too small.

One needs external source to restart the discharge.

Statistical Instability – how it works

- Mapping (fluid theory+fluctuations) $\tau_i << T$, $\bar{x} \equiv x(t+\tau_i)$

$$\bar{q} = q + (\tau_i \kappa / L)p + ae^{-q/2} r \cdot \sqrt{\frac{3}{\langle N_i \rangle \gamma}}$$

$$a \sim 1, \qquad r \in (-1, 1)$$

$$\langle r \rangle = 0, \quad \langle r^2 \rangle = 1/3$$

$$\bar{p} = p + \tau_i \lambda (1 - e^{\bar{q}})$$

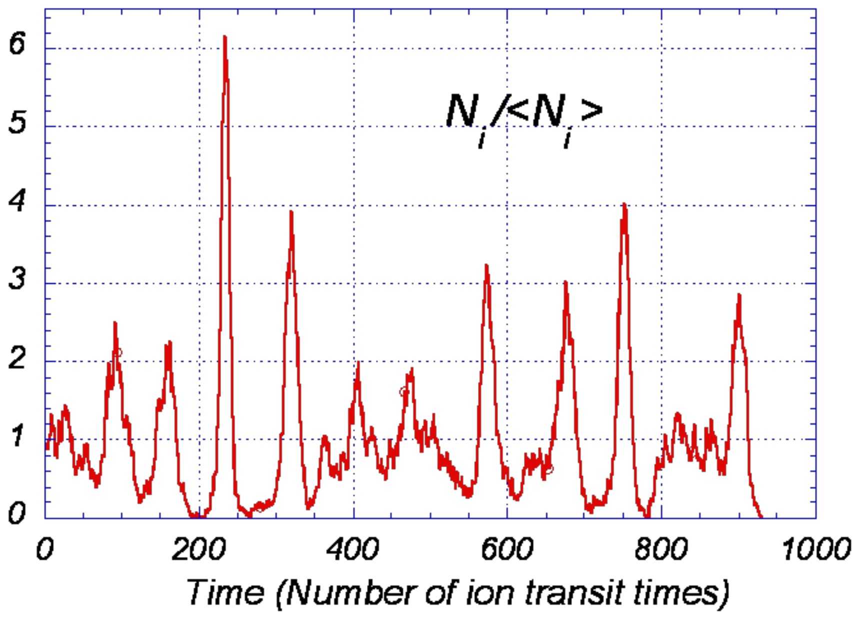
Energy conservation,
when no fluctuations:

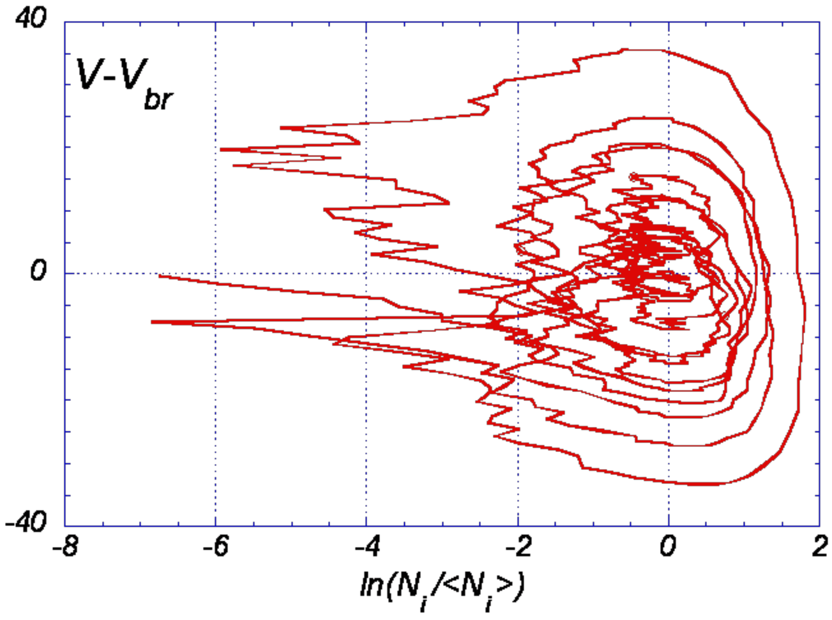
$$a = 0 \Rightarrow J = \frac{\partial(\bar{p}, \bar{q})}{\partial(p, q)} = 1$$

Energy fluctuates when
Fluctuations are present:

$$a \neq 0 \Rightarrow J = -ae^{-q/2}(r/2)\sqrt{\frac{3}{\langle N_i \rangle \gamma}}$$

Statistical Instability – how it works

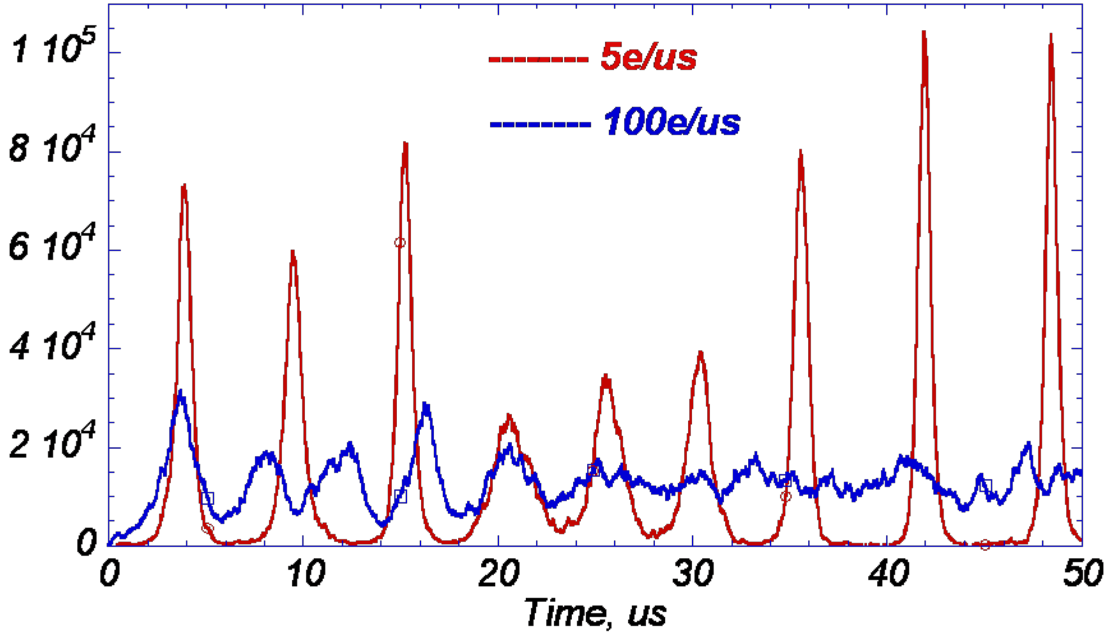




This simple model correctly (qualitatively) describes the instability

Discharge with external source at the cathode

- Exoemission (1D test cell – initially $n_e(0)=n_i(0)=0$, 3D PIC/MC, *$\langle N_i \rangle$=15000, γ_{Xe}= 0.001, mix 93%Ne+7%Xe*)

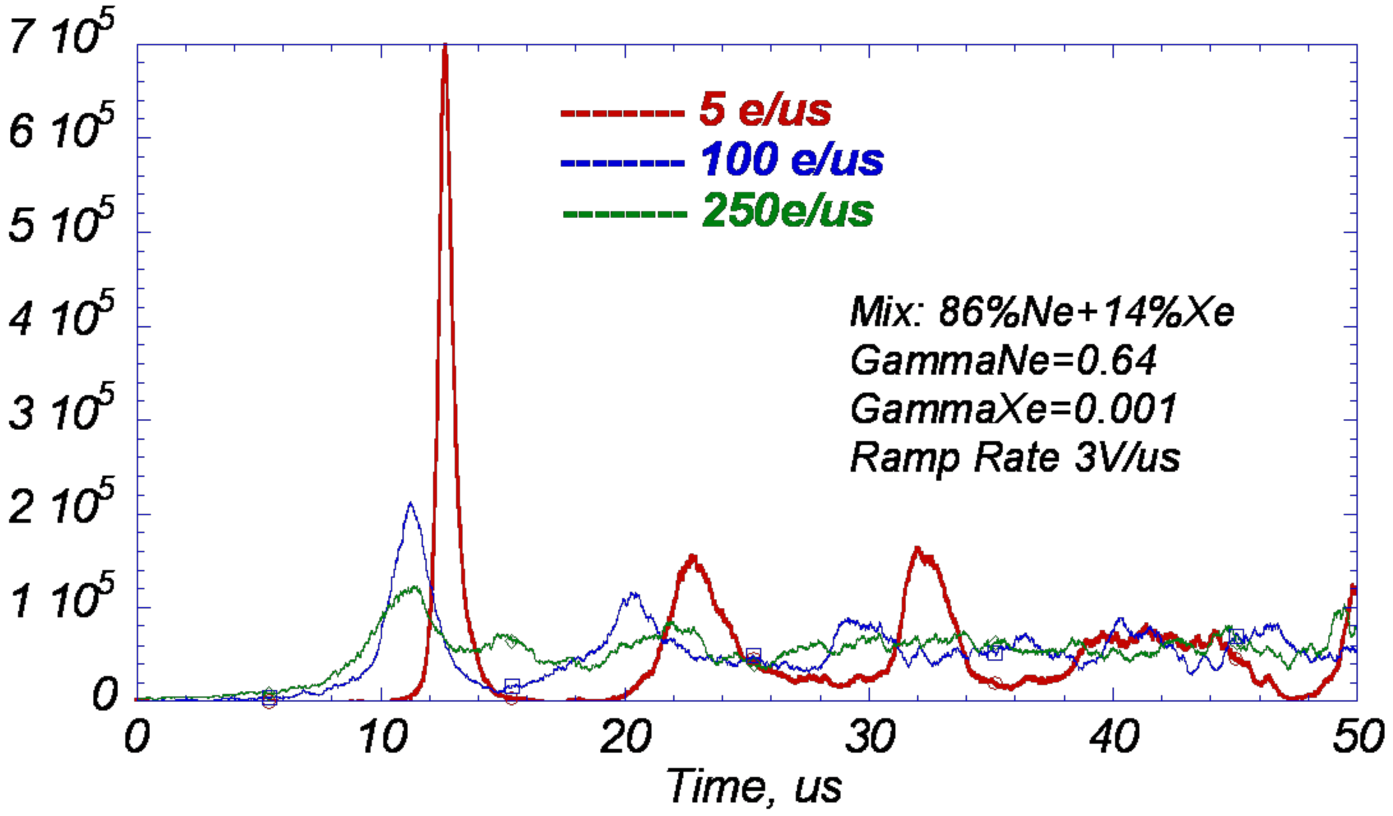


Weak source results in separate peaks, strong source – stable discharge

Discharge with external source at the cathode

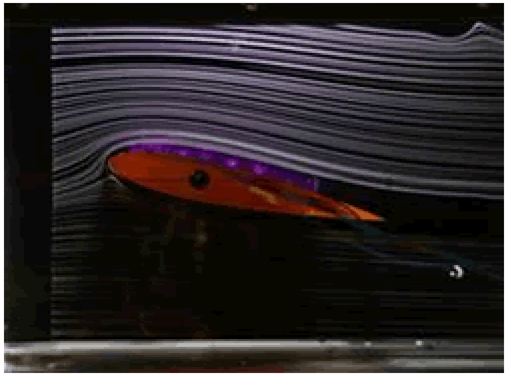
- Exoemission - 3D cell initial conditions ($n_e(0)=n_i(0)=0$),

  3D PIC/MC, $< N_i > \sim 60000$



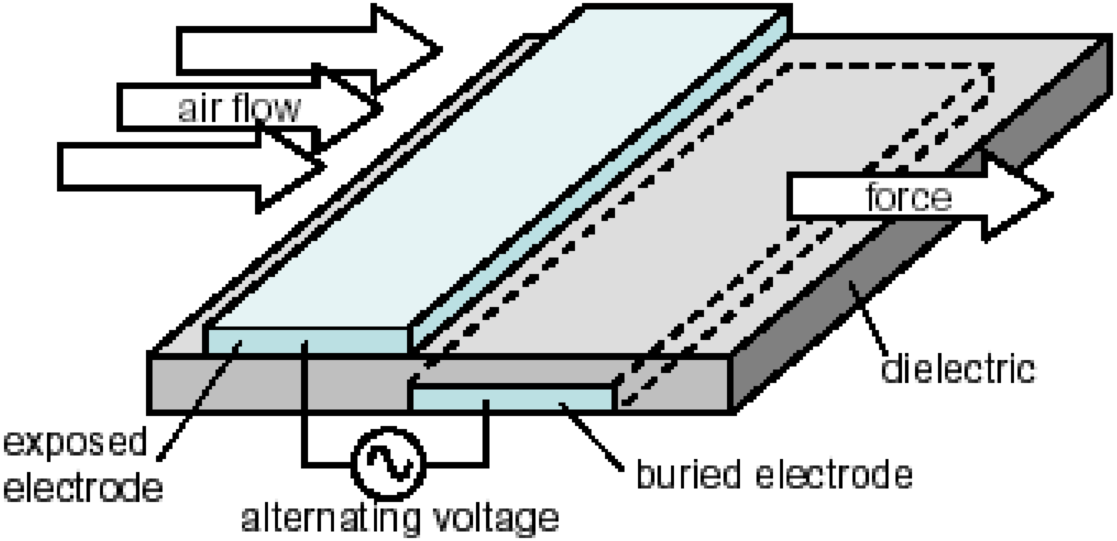
Possibly Statistical instability in a Macro-system: Plasma actuator

OFF



ON

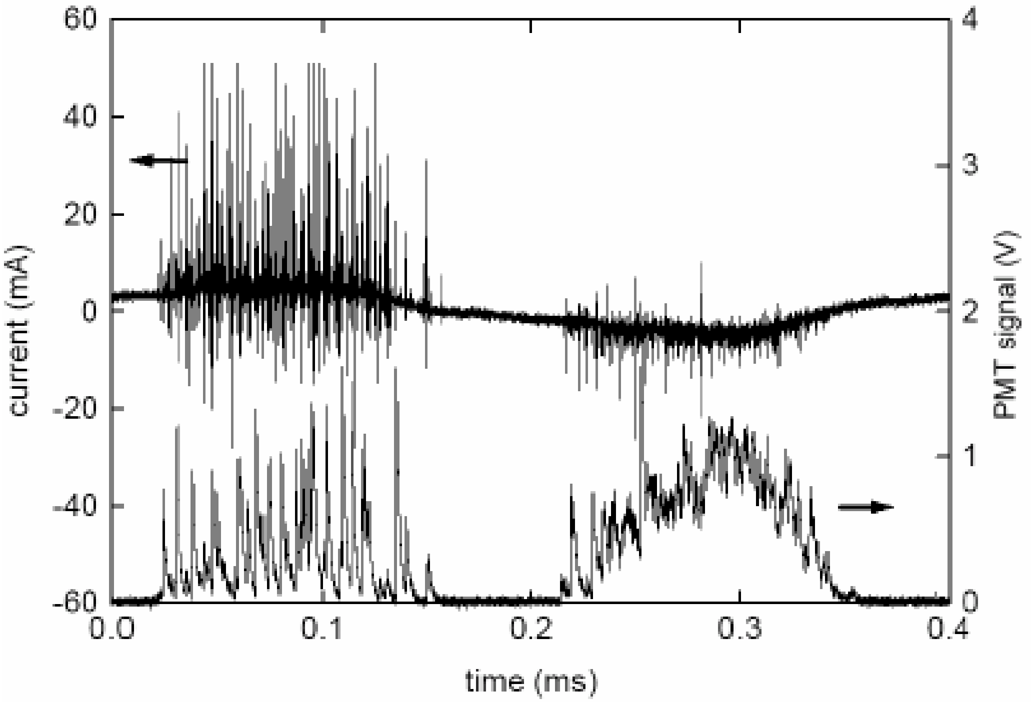
These two pictures are from www.agt.com



This picture from G.I. Font

Possibly Statistical instability in a Macro-system: Plasma actuator

- **Experiment** [Enloe, et al.]: both current through electrodes and PMT signal show clearly statistical instability in the presence of sources, if one assumes that coming air has some level of ionization (Saha or some other equilibrium).



Potential of exposed electrode

Grows Falls

First part – exposed electrode works as the anode – source is very weak (mostly secondary emission from dielectric), the second part it works as a cathode – source is strong (both electrons from the air and secondary emission from the metal).

Summary

- **New kind of instability** is observed in 3D PIC/MC simulations of a dielectric barrier microdischarge – ramp discharge in a PDP cell. The origin of this instability is in dual nature of the Townsend discharge – macroscopic and microscopic. The **macroscopic (**fluid) nature of the Townsend discharge is responsible for oscillations, and amplification of the fluctuations, which come from the second - **microscopic** nature of this discharge - statistics of the ionization and secondary emission. Shown that the value of the secondary emission coefficient is critical.

- It is shown that Townsend discharge in a PDP cell is unstable toward destruction, due to statistical instability.

- External sources may stabilize the discharge. Ramp discharge measurements based on LINE current or/and light integration, and fluid-like simulations miss the statistical part of the ramp discharge behavior. Experiments on large cell also miss statistical effects (very large N). With larger effective γ, the required level of external source may be smaller.

- Instability may be important even for a macrosystem, if its elements are isolated.

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Ramp

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